

THE ^{531.6. 7276}
DESCRIPTION, NATURE and
GENERAL USE, ^{K. S. L.}
OF THE
SECTOR
AND
PLAIN-SCALE,

Briefly and plainly laid down.

AS ALSO A
SHORT ACCOUNT
OF THE
USES
OF THE
*Lines of Numbers, Artificial Sines
and Tangents.*

*London: Printed for Tho. Wright; and sold by
Tho. Heath Mathematical Instrument Ma-
ker, next the Fountain Tavern in the Strand.*
1721.

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P R E F A C E.

Something concerning the Description and Use of the Sector and Plain-Scale being at this time very much wanted; and so I presume will be very acceptable: therefore I have complied this small Treatise of them, in which is briefly contained, their Description, Nature, and General Use.

In the first Chapter is contained, not only the Description of the Lines upon the Plain-Scale, but likewise their Construction and Nature are therein shewn: As also a short Account of their Use.

In the second Chapter is contained, the Description of the Sector, and the Lines now commonly placed thereon.

In the third Chapter is shewn, the Ground and General Use of the Sector. From this Chapter it appears, that the Sector serves as a Scale to all Radius's, not greater than its Length, when quite opened; or lesser than the Distances from 90 to 90 of the Sines, 60 and 60 of Chords, or 45 and 45 of Tangents, when the Sector is quite shut: And so by this Instrument
the

The Preface

The Sphere may readily be projected, upon the Plan of any Circle whose Radius not exceeds the afore-prescribed Limits. Whence the Excellency of this Instrument above the Plain-Scale manifestly appears, for that is a Scale but to one Radius; and therefore by it, it is not easie to project the Sphere, unless upon a Circle whose Radius is of a given Length.

In the fourth Chapter, you have the chief particular Uses of the Lines of Lines, and the Lines of Polygons.

In the fifth Chapter, is shewn the manner of working Proportions with Lines and Sines, Sines and Tangents, Lines and Tangents, &c. As also the excellent Use of the Sector in drawing the Hour Lines upon an horizontal and erect South Plan.

Lastly, in the sixth Chapter is contained, the Use of the Lines of Numbers, artificial Sines, and Tangents. What is said in this Chapter, I am of Opinion, is sufficient to shew Persons indifferently skilled in Arithmetick and Geometry the Manner of using those Lines.

Thus, courteous Reader, have I given you the Contents of what is to be seen in the following little Tract, hoping you will candidly receive the same. And so I remain, yours, &c.

E. S.

THE



T H E
Description, Nature and General Use,
O F T H E
Sector and Plain-Scale, &c.



C H A P. I.
*Concerning Definitions of Chords, Sines, Tan-
gents, &c. and the Manner of Projecting
them, and putting them on Rulers, commonly
call'd Plain-Scales.*

 Ecause of the six Parts of a Trian-
gle, *viz.* the three Sides and the
three Angles, any three of them
being given, independent of one
another, the other three are there-
by limited, and consequently some way or
other to be found. And since the Angles of
a Triangle, that is, the Arcs of equal Circles
described about the angular Points, and com-
prehended between the Sides, (which are pro-
portional to the Angles, from *prop. 33. lib. 3.*
B *Euclid*

Euclid) are not proportional to its Sides, the Ancients devised certain right Lines, appertaining to a Circle, which come in competition with Angles or Arcs. These right Lines are thus defined;

A Chord of an Arc or Angle, is a right Line drawn from one end of an Arc to the other; as the Line AB is the Chord of the Arc AB; it is also the Chord of the Arc AFB, because it is common to both Arcs of the Circle.

Whence it is manifest, (*per prop. 15. lib. 3. Euclid*) that the greatest Chord that can be drawn in a Circle is its Diameter, or the Chord of 180 Degrees, or a Semicircle: Whence all Chords of Arcs, greater than a Semicircle, are lesser than the Diameter.

A right Sine of an Arc, is a right Line drawn from one end of that Arc, perpendicular to a Diameter drawn to the other end of it, and belongs to both Arcs of a Semicircle; as BC is the right Sine of the Arc AB, and also of the Arc FB. Hence it is manifest, from the aforecited Proposition of *Euclid*, that the greatest Sine is equal to the Semidiameter GE, or the Sine of 90 Degrees: Therefore all Sines of Arcs greater than 90 Degrees, or a Quadrant, are lesser than the Semidiameter, or Radius.

The versed Sine of an Arc, is that Portion of the Diameter included between the right Sine of the said Arc, and the Arc it self. As AC is the versed Sine of the Arc AB, and EC the

the versed Sine of the Arc FGB; hence the greatest versed Sine is the Diameter AF, viz. the versed Sine of 180 Degrees, or a Semicircle.

A Tangent of an Arc, is a right Line touching the Circumference of a Circle, and (from *prop. 16. lib. 3. Euclid*) is at right Angles to the Diameter drawn thro' the Point of Contact, and limited by the Secant of the same Arc; as AD is the Tangent of the Arc AB; whence there can be no Tangent of 90 Degrees, or a Quadrant, for then the Tangent will be infinitely long.

The Secant of an Arc, is a right Line drawn from the Center of a Circle, thro' one end of the Arc it is the Secant of, till it meets the Tangent, raised at right Angles to a Diameter drawn to the other end of the Arc; as ED is the Secant of the Arc AB; whence there can be no Secant of 90 Degrees, or a Quadrant, because then the Secant will be infinitely long.

The Half or Semitangent of an Arc, is that Portion, next to the Center, of a right Line drawn from the Center of the Circle parallel to the Tangent of the Arc, cut off by the Chord of the Complement of the Arc to 180 Degrees, as EH is the half Tangent of the Arc AB. But because (from *prop. 20. lib. 3. Euclid*) the Angle BFA is $\frac{1}{2}$ of the Angle BEA, the Side FE equal to the Side EA, and the Angles EAD, FEH right ones, HE will be equal to the Tangent of half the Arc BA; that is, the

Semitangent of any Arc is but the Tangent of half that Arc; whence the greatest Semitangent, or that of 180 Degrees, is infinite.

The Circumference of every Circle is supposed to be divided into 360 equal Parts, called Degrees; and each Degree in 60 equal Parts, called Minutes, &c. This Number was chosen by Geometricians for the Division of a Circle, because it may be divided into a greater Number of Parts without any Remainder, than any lesser Number than 360.

The Complement of an Arc, or Angle, is what it wants of a Quadrant, or 90 Degrees, or of a Semicircle, or 180 Degrees; or lastly, of a whole Circle, or 360 Degrees. As 20 Degrees is the Complement of 70 Degrees to a Quadrant, because 20 Degrees is the Remainder of 70 Degrees subtracted from 90 Degrees, likewise 50 Degrees is the Complement of 130 Degrees to 180 Degrees, and 70 Degrees the Complement of 290 Degrees.

How to Project the Plain-Scale.

First draw a Circle, ABDC, which *Fig. 2.* cross at right Angles with the Diameters AD, CB; then continue out AD to G, and upon the Point B raise BF perpendicular to CB. Now draw the Chord AB, and divide the Quadrant AB into 9 equal Parts, setting the Figures 10, 20, 30, &c. to 90, to them; each of which 9 Parts again subdivide into 10 equal Parts, and then the Quadrant

rant will be divided into 90 Degrees. Now setting one Foot of your Compasses in the Point A, transfer the said Divisions to the Chord Line AB, and set thereto the Figures 10, 20, 30, &c. and the Line of Chords AB, will be divided, and then may be put upon your Scale.

Now to project the Sines, divide the Arc BD into 90 Degrees, as before you did AB; from every of which Degrees let fall Perpendiculars on the Semidiameter EB, which Perpendiculars will divide EB into a Line of Sines, to which you must set the Numbers 10, 20, &c. beginning from the Center, and then you may transfer the Line of Sines to your Scale.

Again, to project the Line of Tangents, from the Center E, and thro' every Division of the Arc BD, draw right Lines cutting BF, which will divide it into a Line of Tangents, setting thereto the Numbes 10, 20, 30, &c. which you must transfer to your Scale.

To project the Line of Secants, transfer the Distances E 10, E 20, E 30, &c. that is, the Distance from E to 10, 20, 30, &c. on the Tangent Line, upon the Line EG, and setting thereto the Numbers 10, 20, 30, &c. the Line EG will be divided into a Line of Secants.

To project the Semitangents, draw Lines from the Point C thro' every Degree of the Quadrant AB, and they will divide the Semidiameter AE into a Line of Semitangents: But because the Semitangents on Scales run to 160 Degrees, continue out the Line AE, and draw
Lines

Lines from the Point C, thro' the Degrees of the Quadrant CA, cutting the said continued Portion of AE, and you will have a Line of half Tangents to 160 Degrees, or farther if you please.

Moreover to draw the Rhumb Line, from every 8th Part of the Quadrant AC, setting one Foot of your Compasses in A, describe Arcs cutting the Chord AC, which will divide AC into a Line of whole Rhumbs; and after the same manner may the Subdivisions of half and quarter Rhumbs be made.

Lastly, to project the Line of Longitude, draw the Line HD equal and parallel to the Radius CE, which divide into 60 equal Parts, every 10 of which Number. Now from every of those Parts let fall Perpendiculars to CE, cutting the Arc CD, and having drawn the Chord CD with one Foot of your Compasses in D, transfer the Distances from D to each of the Points in the Arc CD, on the Chord CD, and set thereto the Numbers 10, 20, &c. and the Line of Longitude will be divided.

These be all the Lines commonly put on one side of the Plain-Scale, excepting equal Parts, which want no description: And on the other side is a Decimal or Diagonal Scale, on which an Inch, or some part thereof, as $\frac{1}{2}$, or $\frac{1}{4}$, is divided into 100 equal Parts, by means of Diagonals, in the following manner;
Fig. 3. Suppose AB to be the Length of the Scale, which let be 6 Inches; then
 having

having chosen a convenient Breadth, as BC , make the Parallelogram $AEBC$, and draw Lines from the Division of every Inch, parallel to BC ; then divide an Inch, suppose DB , into 10 equal Parts, as also FC , and draw the Diagonals D 10, 10 20, 20 30, &c. Likewise divide BC , and AE , each into 10 equal Parts, thro' which draw 10 Parallels, and set the Numbers as *per* Figure, and your Scale will be made. After the same manner may one half or one quarter of an Inch be decimally divided.

Now if you have a mind to take any part of an Inch, supposed to be divided into 100; as the 48th part, look for the Division 40 on the Line DB , and guiding your Eye along its Diagonal, mark where the 8th Parallel to BD cuts the said Diagonal, and the Distance of that Point of Section from the Line DF , will be the 48th part of an Inch. Again, to find the 100th part of an Inch, look where the Parallel 1 cuts the Diagonal D 10, and the Distance from that Point to DF will be the 100th part of an Inch. For from the Similiarity of Triangles, demonstrated *per prop. 4. lib. 6. Euclid*, as DF is to $\frac{1}{10}$ of DB , or 10 F , so is $\frac{1}{10}$ of DF to $\frac{1}{100}$ of DB .

Hence it is manifest, that by means of this Scale, any Line may be drawn whose Length is expressible in less than four Figures, whether they be whole Numbers, or mixed, that is decimal Fractions, and whole Numbers; as if the Length of the Line be 365; then you must

must call 1 Inch 100, and so 3 Inches will be 300. Then the 65 must be taken from DF, to the Intersection of the 60th Diagonal with the 5th Parallel, and the Extent from 3 Inches to that Point of Intersection, will give the Length of the Line expressible by 365. Likewise the same Extent may be taken for the Length of a Line expressible by 36.5, in supposing 1 Inch to be 10. Also it may be expressed by 3.65, in taking 1 Inch for 1. And lastly, it may be expressed by .365, in taking 1 Inch for $\frac{1}{10}$. And this is the Use of the Diagonal Scale.

As for the Uses of the Chords, Sines, Tangents, &c. the Chords are to lay off the Quantity of any Angle desired upon a Point given, in a right Line. And contrarywise, to measure the Quantity of an Angle already laid down. The first is done by always taking the Extent of 60 Degrees of Chords between your Compasses, and describing an Arc about the angular Point; and then laying off the Number of Degrees proposed, upon the said Arc, and drawing a right Line from the angular Point. And the latter, by always making an Arc of 60 Degrees of Chords about the angular Points, and then taking the Chord of the said Arc between your Compasses, and measuring it on the Line of Chords.

Example, To make an Angle of 30
Fig. 4. Degrees on the Point A, take 60 Degrees of Chords in your Compasses, and setting one Foot of your Compasses in the Point

Point A, describe the Arc DC; then take 30 Degrees from your Chords, and lay them off from D to C, and draw the Line AC, and the Angle CAB will be 30 Degrees.

Now to measure an Angle, suppose CAB, take 60 Degrees of Chords between your Compasses, and setting one Foot in the Point A, describe the Arc CD; then take between your Compasses the Distance from C to D, which measur'd on the Chords will be found to reach to 30 Degrees, the Quantity of the Angle sought.

The Sines are to orthographically project the Sphere, &c.

The Tangents, half Tangents and Secants, are used in finding the Centers, and Poles, of projected Circles, in the Stereographical Projection of the Sphere, &c.

The Rhumbs, are to lay down the Angles of a Ship's Way in Navigation.

And the Line of Longitude determines by Inspection, how many Miles there are in a Degree of Longitude, in each several Latitude: As, in the Latitude of no Degrees, that is, under the Equator, 60 Miles make a Degree; in the Latitude of 40 Degrees, 46 Miles make a Degree; in the the Latitude of 60 Degrees, 30 Miles make a Degree; in the Latitude of 80 Degrees, 10 Miles make a Degree.

C H A P. II.

The Description of the Sector, and the Lines generally placcd thereon.

THE *Sector* is a Mathematical Instrument, consisting of two equal Legs, of Brass, Silver, Ivory or Wood, moveable about a Joynt, like a Carpenters Rule; so that the said Legs, together with certain right Lines, drawn from the Center of the Joynt, contains Angles of different Magnitudes. The Length of these Legs are not determined, but yet they are generally about 6 Inches, having a proper Breadth and Thickness. This Instrument takes its Name, from a Sector in Geomotry, which *Euclid* defines, in the 9th Def. of his 3^d Book, to be a Figure comprehended under two right Lines making an Angle, and the Arc of a Circle described about the angular Point, intercepted between the two said right Lines.

The right Lines that be commonly drawn on the Faces of this Instrument, to be used Sectorwise, are six, *viz.* The Lines of Lines, or equal Parts, the Lines of Chords, the Lines of Sines, the Lines of Tangents, the Lines of Secants, and the Lines of Poligons.

The Lines of equal Parts are two equal right Lines drawn upon the Legs of the Sector, on the same Side, from the Center of the Joynt, almost to the Ends of the Legs, making unequal Angles with the inward Edges of their respective Legs; that so, the Lines
of

This image displays a handwritten musical score, likely for a string quartet, consisting of four systems of staves. The notation is in a historical style, featuring various musical symbols, clefs, and performance markings.

The first system includes a treble staff with a key signature of one flat (B-flat) and a common time signature (C). The lower staves are marked with "60C" and "45T".

The second system features a treble staff with a key signature of one flat (B-flat) and a common time signature (C). The lower staves are marked with "60C" and "45T".

The third system includes a treble staff with a key signature of one flat (B-flat) and a common time signature (C). The lower staves are marked with "60C" and "45T".

The fourth system includes a treble staff with a key signature of one flat (B-flat) and a common time signature (C). The lower staves are marked with "60C" and "45T".

Performance markings such as "tan." (tutti) and "P" (piano) are visible throughout the score. The notation includes various musical symbols, clefs, and performance markings, suggesting a complex and expressive piece of music.

of Sines, and Secants, which are likewise drawn upon the same Faces of the Legs from the Center of the Joynt, may make equal Angles with each other, and likewise equal to that which the said Lines of Lines make. These Lines of Lines, are each divided into 100 equal Parts, each of which are again subdivided into Halfs, and Quarters. If the Length of the Legs, are sufficient. The Divisions are number'd, from the Center, with 1, 2, 3, &c. to 10. near which are placed the Letters LL. The three Parallels, that be drawn, to each of these Lines, are for the Eyes better distinguishing the Divisions, and Subdivisions, as also are those near the other Lines.

The Lines of Sines, are two equal Lines of natural Sines, drawn upon the same Faces of the two Legs of the Sector from the Center of the Joynt, as the Lines are, of a Radius, equal in Length to either of the Lines of Lines. These are number'd with the Figures 10, 20, &c. to 90. at the Ends of which stand the Letters SS.

The Lines of Chords, on the other Faces of the same Legs, are two natural Lines of Chords (made as directed in the last Chapter) drawn from the Center of the Joynt upon each Leg, so as to make an Angle with each other, equal to the Angle that the Lines of Sines, &c. make. The Radius of these Lines of Chords is equal in length to the Radius of the Lines of Sines. They are number'd with 10, 20, &c. to 60

Degrees; the Chord of which, *per Corol. prop. 15. lib. 4. Euclid*, is equal to the Radius, These Lines are denoted by the Letters CC.

The two Lines of Tangents, are Lines of natural Tangents, (divided as directed in the last Chapter) whose Radius is equal in length to the Radius of the Lines of Sines. These are drawn from the Center of the Joynt on each Leg, making the same Angle with each other, as the Sines, &c. do. Each of these Lines of Tangents run but to 45 Degrees, and are number'd accordingly. The reason of this is, because the Tangent of 45 Degrees is equal to Radius; that so when the Sector is set to 90 Degrees on the Lines of Sines, the same Extent, that is, from 90 Degrees on one Line of Sines to 90 Degrees on the other, will reach from 45 Degrees to 45 Degrees on the Lines of Tangents; as also, from 60 Degrees to 60 Degrees, on the Lines of Chords. The Lines of Tangents are marked with the Letters TT.

There are also two Lines of lesser Tangents, whose Radius is about two Inches in length, drawn from the Center of the Joynt, and making an Angle with each other, equal to the Angle that any one of the aforesaid Lines make respectively with each other; tho' on some Sectors this Angle is lesser. These Lines of Tangents are number'd from 45 Degrees, which stands at the Ends of the Radius from the Center of the Joynt, to about 75 Degrees. They

They have the Letters *tt* set to them, and serve to supply the Defect in the other Lines of Tangents.

The Lines of Secants, are divided from a Circle of the same Radius as the Lines of lesser Tangents are, and make the same Angle with each other as the lesser Tangents do. These begin at the Length of the Radius from the Center, and run to about 75 Degrees. The Numbers 10, 20, 30, &c. to 75, are set to them, and they are known by the Letters *SS*.

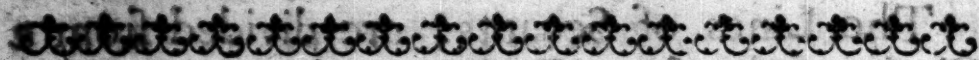
Lastly, The Lines of Polygons, are drawn from the Center of the Joynt very near the inward Edges, and number'd from 4, 5, &c. to 12, which is about 3 Inches from the Center. These Lines are denoted with the Letters *PP*.

Thus have I described all the Lines that be commonly drawn on a Sector to be used Sectorwise.

The other Lines that be arbitrarily placed, near and parallel to the outward Edges of the Sector, on both Faces thereof, and which are to be used, as on common Scales, are the Lines of artificial Numbers, Sines and Tangents; a Foot divided into 12 Inches, and each Inch subdivided into 20 equal Parts. Also sometimes the Lines of Latitude, Hours, and Inclination of Meridians are put on it.

Note, The artificial Numbers, are denoted with the Letter *N*, the Sines with the Letter *S*, and the Tangents with the Letter *T*. When Proportions are to be work'd
by

by these artificial Lines, the Sector must be quite open'd.



C H A P. III.

Of the Foundation and General Use of the Sector.

BECAUSE, from *Prop. 4. lib. 6. Euclid*, the homologous Sides of similar or equiangular Triangles, are proportional to each other; therefore if the Lines AB, AC, represent, for example, the two Lines of Lines, making any Angle BAC with each other. Now if the Line BC be drawn, and Aa, Ab, taken equal, and the Line ab drawn, AB, or AC, will be to BC, as Aa is to ab. Whence if the Line AB or AC be double, triple, quadruple, &c. the Line Aa or Ab, the Line BC will be double, triple, quadruple, &c. of the Line ab. As suppose AB, or AC, be 100, and Aa, or Ab 50, and BC 60, then ab will be 30; for as AB 100 is to Aa 50, so is BC 60 to ab 30. And the like Reason holds for all other Proportions.

Whence if the Lines AB, AC, represent the Lines of Chords, Sines or Tangents; that is, if the Lines AB, or AC, be the Radius, and the Line Aa is the Chord, Sine or Tangent of a proposed Number of Degrees, to the said Radius AB, the Line ab will be the Chord, Sine

or

or Tangent of the same Number of Degrees to the Radius BC.

For example, To find the Chord, Sine or Tangent of 15 Degrees, to a Radius of 4 Inches. Open the Sector so, that the Distance from 60 to 60 on the Lines of Chords be 4 Inches; then the Distance between 15 and 15 on the Lines of Chords, will be the Chord of 15 Degrees to the Radius of 4 Inches. The Sector still remaining thus opened, the Extent from 15 Degrees to 15 Degrees on the Lines of Sines, will be the Sine of 15 Degrees, to the aforesaid Radius of 4 Inches. Also the Extent from 15 Degrees to 15 Degrees on the Lines of Tangents, will give the Tangent of 15 Degrees to the aforesaid Radius.

But if the Chord propos'd be more than 60 Degrees, as suppose the Chord of 80 Degrees to the aforesaid Radius be requir'd, you must take, as before directed, the Chord of half 80 Degrees, *viz.* 40 Degrees, *Fig. 6.* and having described an Arc, AD, with the said Radius, lay off the Chord of 40 twice; to wit, from A to B, and from B to C; draw the Line AC, which will be the Chord of 80 Degrees.

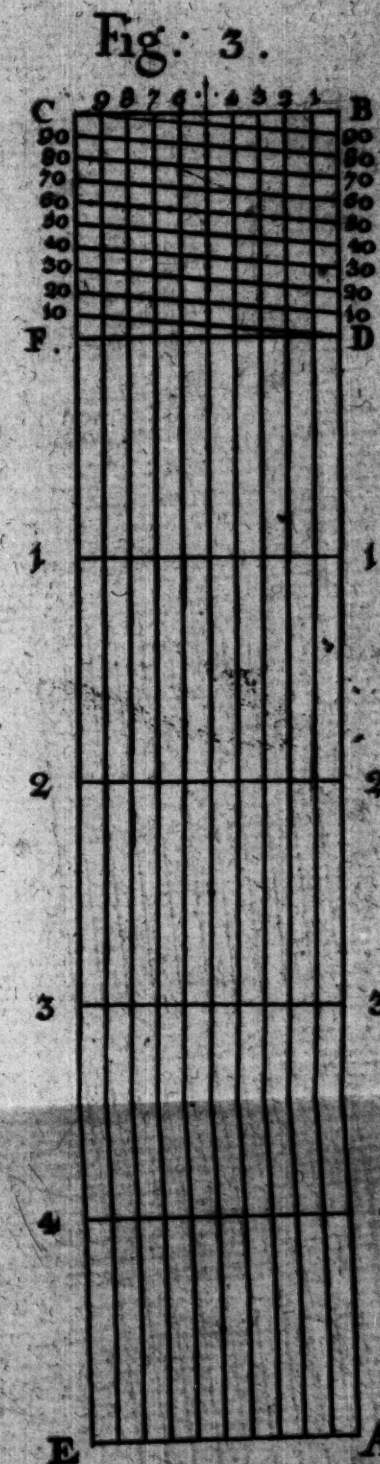
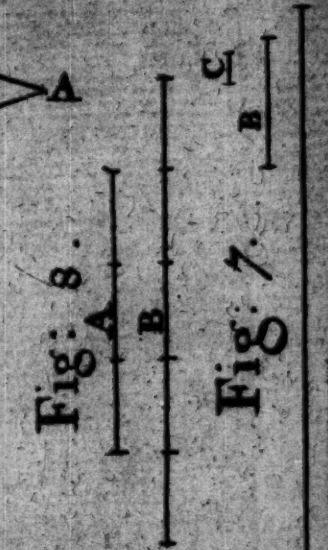
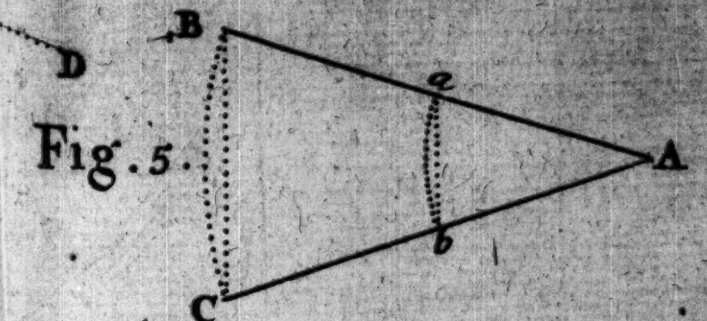
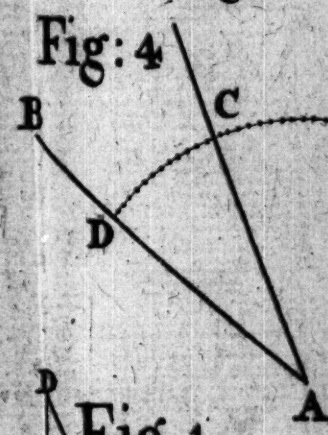
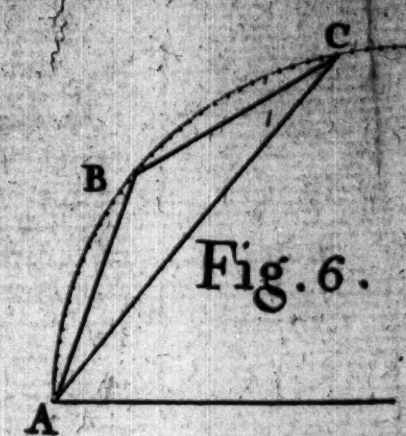
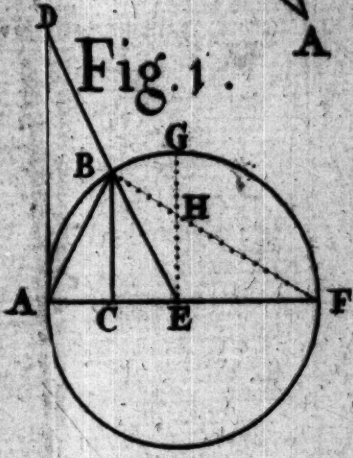
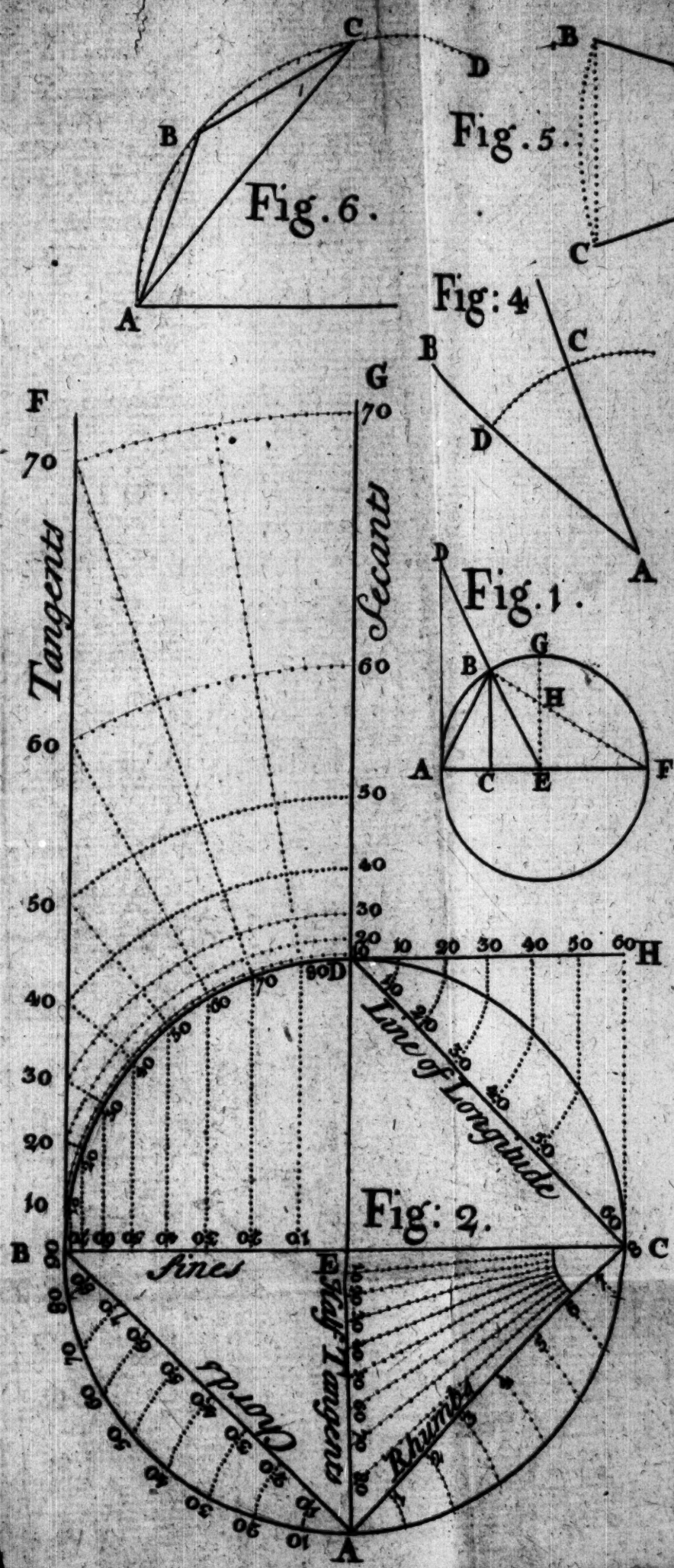
If the Tangent of a Number of Degrees greater than 45, suppose 74 Degrees, be sought to a Radius of 3½ Inches, you must make use of the Lines of lesser Tangents; because the greater ones run but to 45 Degrees, thus. Open the Sector so, that the Distance between

45 Degrees and 45 Degrees of the lesser Tangents, be equal to $3\frac{1}{2}$ Inches; then the Distance from 74 Degrees to 74 Degrees, will be the Tangent of 74 Degrees to a Radius of $3\frac{1}{2}$ Inches.

If the Secant of a proposed Number of Degrees to a given Radius be sought; as suppose the Secant of 50 Degrees to a Radius of 1 Inch be required, open the Sector so, that the Distance between 0 and 0, on the Lines of Secants be 1 Inch; then the Distance from 50 Degrees to 50 Degrees on the same Lines, will be the Secant of 50 Degrees.

If the Converse of any of these Operations be requir'd; that is, if a given right Line be the Chord, Sine, Tangent or Secant of a proposed Number of Degrees, suppose 16, and the Radius to it be sought, open the Sector so, that the Distance between 16 and 16, on the Lines of Chords, Sines, Tangents or Secants, be equal to the Length of the given Line, according as it be a Chord, Sine, Tangent or Secant. Then the Distance between 60 Degrees and 60 Degrees on the Lines of Chords; between 90 Degrees and 90 Degrees on the Lines of Sines; between 45 Degrees and 45 Degrees on the Lines of Tangents; and between 0 Degrees and 0 Degrees on the Lines of Secants, will be the Radius to the said right Line of 16 Degrees, according as it is a Chord, Sine, Tangent or Secant.

From what hath been already said, it appears that the Sector is as it were a universal Plain-



Plain-Scale; for whereas on the Plain-Scale, the Chords, Sines, Tangents and Secants, have but one Radius; so by this Contrivance, the Chords, Sines, Tangents or Secants to all Radius's may be had. But with this Limitation, that the Radius given be not longer than the Length of twice the Radius of the respective Chords, Sines, Tangents or Secants, on the Legs of the Sector; or shorter than the Distance from 60 to 60 of Chords, 90 and 90 of Sines, or 45 and 45 of Tangents, when the Sector is quite shut.

Moreover, Because the Lines of Lines, Chords, Sines, Tangents and Secants, make equal Angles with each other respectively, therefore Proportions in equal Parts and Sines, equal Parts and Tangents, Sines and Tangent, &c. by help of a Pair of Compasses, and the Sector, may be work'd. And so the Cases of Plain and Spherical Trigonometry may, by means of the Sector, be solved. An Instance or two of which I shall give, after having given some of the particular Uses of the Lines of Lines; and the Use of the Lines of Polygons.



D

CHAP.

C H A P. IV.

*Of the particular Uses of the Lines of Lines;
and the Use of the Lines of Polygons.*

Fig. 5. **F**OR the Ease of Expression, in shewing the Use of the Lines on the Sector, the Lines found out by the Sector are said to be lateral or parallel; lateral are those found on the Sides of the Sector, as AB, AC; and parallel are the Lines that run from one Leg of the Sector to the other, equally distant from the Center of the Joynt; as BC, ab.

P R O B. I.

To lay down a Line expressable by any given Parts, or Fractions of Parts.

Either of the Lines of Lines are actually divided into 100 equal Parts, but we have put only 10 Numbers to them, which may signifie either themselves alone, or 10 times themselves, or 100 times themselves, or 1000 times themselves, as occasion requires. As suppose the Numbers given be no more than 10, then we may suppose the Lines only divided into 10 Parts, according to the Numbers set to them. If they be more than 10, but lesser than 100, then either Line will contain 100 Parts, and the Numbers set by them will be in value, 10, 20, 30, &c. as they are actually divided. If yet they be more than 100, then every Part must

must be supposed divided into 10, and either Line will be 1000 Parts, and the Numbers set to them will be in value 100, 200, 300, and so forwards, still encreasing themselves by 10. This being pre-supposed, we may number the Parts, and Fraction of Parts given on either of the Lines of Lines; which Distance taken from them with a Pair of Compasses, and drawn on your Paper, will give the Length of the Line sought.

Example. Suppose a Line is to be drawn representing 75; take 75 of the 100 Parts either of the Lines of Lines is divided into, and note it on A; or take $7\frac{1}{2}$ of the first 10 Parts, and note it on B; or only take $\frac{3}{4}$ of those 100 Parts, and note it in C; either of which will be Lines expressible by 75. *Fig. 7.*

P R O B. II.

To Encrease, or Deminish, a Line in a given Proportion.

Take the Line given between your Compasses, and open the Sector, so as the Feet of the Compasses may stand in the Points of the Number given on each of the Lines of Lines; then keeping the Sector thus opened, the parallel Distance of the Points of the Number required, will give the Line sought.

Example. Let A be a Line given to be encreased in the Proportion of 3 to 5. First take the Line A between your *Fig. 8.*

D 2

Compasses,

Compasses, and open the Sector so, that the Distance from 3, to 3 on the Lines of Lines be equal to that Extent; then the parallel Distance of 5 and 5, will give the Line B, sought.

In the same manner, if B be a Line given to be diminished in the Proportion of 5 to 3, take the Line B between your Compasses, and to it open the Sector in the Points of 5 and 5 on the Lines of Lines; then the parallel Distance between 3 and 3, will give the Line A, required.

If this way of working be not sufficient, you may multiply the Numbers given by 2, 3, 4, &c. and work by their Equimultiples; as, for 3 and 5, you may open the Sector in 6 and 10, 9 and 15, 12 and 20, 15 and 25, or in 18 or 30, &c.

P R O B. III.

To divide a given Line into any Number of equal Parts.

Take the Line given between your Compasses, and open the Sector in the Points of the Parts the given Line is to be divided into; then keeping the Sector at this Angle, the parallel Distance between the Points of 1 and 1, will divide the given Line into the Parts required.

Example. Let AB be a Line given
Fig. 9. to be divided into 8 equal Parts. First take this Line AB between your Compasses, and to it open the Sector in the Points of 8 and 8; then the parallel Distance between
 1 and

1 and 1, gives the Line AC, which will divide AB into 8 equal Parts.

If the Line proposed be too long to be applied to the Legs of the Sector, divide $\frac{1}{2}$ or $\frac{1}{4}$ of it into the Parts proposed; and the Double, or Quadruple of one of those Parts, will divide the whole Line into the Number of Parts proposed.

P R O B. IV.

To find the Proportion between two or more given Lines.

Take the greater of the given Lines between your Compasses, and to it open the Sector in the Points of 100 and 100; then take the lesser Lines severally, and carry them parallel to the greater, till they stay on the same Numbers of the Lines of Lines; and the Numbers whereon they stay, will give their Proportion to 100.

Example. Let the Lines given be AB, CD. First take the Line CD *Fig. 10.* between you Compasses, and to it open the Sector in the Points of 100 and 100; then keeping the Sector thus opened, enter the lesser Line AB parallel to the former, and you will find it cross the Lines of Lines in the Points 60 and 60; wherefore the Proportion of AB to CD, is as 60 to 100.

Or, if the Line CD be so long that it cannot be apply'd to the Points 100 and 100, then you may suppose the lesser Line AB, to be 100, and cutting off CE equal to AB, you will find the

the Proportion of CE to ED, to be as 100 to almost 67; wherefore this way the Proportion of AB to CD, is as 100 to almost 167.

This Proportion may likewise be worked by any other Numbers that admit several Divisions, as by the Numbers 60 and 60; and then the lesser Number will be found 36, which is as before in lesser Numbers, as 3 to 5. This Proportion may also be worked without opening the Sector, For if the Lines between which a Proportion be sought, are laterally apply'd on the Lines of Lines, (or any other Scale of equal Parts) the Proportion between them will be the same, as between the Lines to which they are equal.

P R O B. V.

To find a Third Proportional to two given Lines.

First place both the given Lines on both sides of the Sector from the Center, and mark the Terms of their Extension; then take out the second given Line again, and to it open the Sector in the Term of the first Line; then keeping the Sector at this Angle, the parallel Distance between the Terms of the second Line, will be the third Proportional sought.

Example. Let the two Lines given be AB, AC, which take out and place on both sides of the Sector, so as they meet in the Center A: let the Terms of the first Line be B, B, and the Terms of the second

second C, C. Then take out AC, the second Line, again; and to it open the Sector in the Terms B, B; so the Parallel between C, C, gives the third Proportional sought. For as AB is to AC, so is BB equal to AC, to CC.

P R O B. VI.

Three Lines being given, to find a fourth Line proportional to them.

Place the first and third Lines on both sides of the Sector from the Center; then take out the second Line, and to it open the Sector in the Terms of the first Line. Now keeping the Sector at this Angle, the parallel Distance between the Terms of the third Line, will be the fourth Proportional sought.

Example. First take out A and C, and place them on both sides of the *Fig. 12* Sector, in AB, AC, and AD, AE, laying the beginning of both Lines in the Center A; then take out B, the second Line, and to it open the Sector in B, C, the Terms of the first Line; and then the Parallel between D and E, gives the fourth Proportional sought.

As in Arithmetick, it is sufficient for the first and third given Numbers to be of one Denomination, and the second and the fourth required to be of another. For one and the same Denomination is not necessarily required in them all. So in Geometry, it is sufficient if the Sides AB, AD, resembling the first and third Lines be measured by one Scale, and the Parallels

parallel BC, DE, be measured in another. Wherefore the Proportion of A, the first Line to C the third, being found by *Prop. 4.* aforegoing, which suppose to be as 8 to 12, or in lesser Numbers, as 4 to 6, or as 2 to 3, or in greater Numbers, as 16 to 24, or 18 to 27, or 20 to 30, 30 to 45, or 40 to 60, &c. If the Sector be opened in the Points of 8 and 8, to the Quantity of B, the second given Line; then the Parallel between 12 and 12, will give DE, the fourth Line required. So likewise if it be opened in 4 and 4, the Parallel between 6 and 6; or if in 16 and 16, the Parallel between 24 and 24, will give the same, DE, and so of the rest.

P R O B. VII.

Two Lines being given, to find a Mean Proportional between them.

Take the longest Line and lay it on the Line of Lines from the Center, noting its Extention; then take the shortest Line between your Compasses, and applying it parallelwise on the Lines of Lines, open the Sector so, that the lateral Distance from the Center of the Sector to the Term of this Line, be equal to the parallel Extent of the Terms of the longest Line; and then the said lateral Distance of the Terms of the shortest Line, or the parallel Distance of the Terms of the longest Line, will give the Mean Proportional sought.

Note,

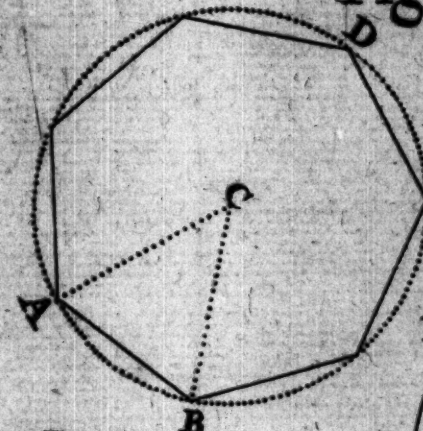
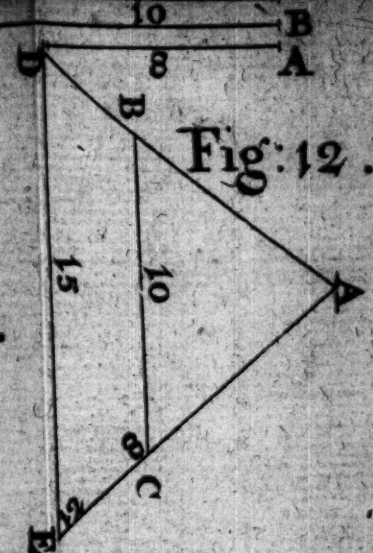
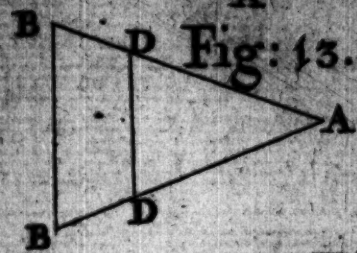


Fig: 16.

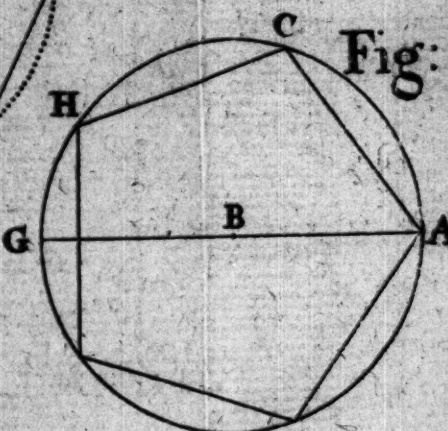


Fig: 15

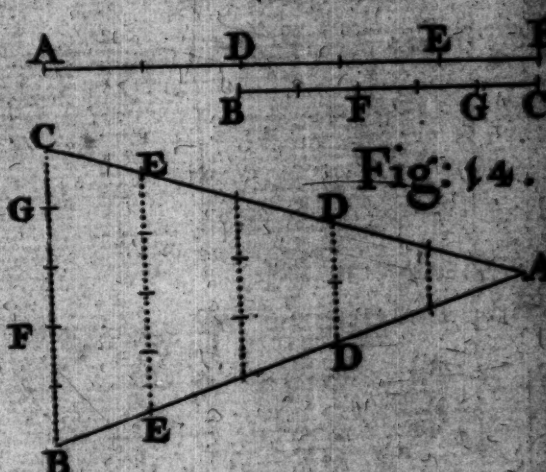


Fig: 14.

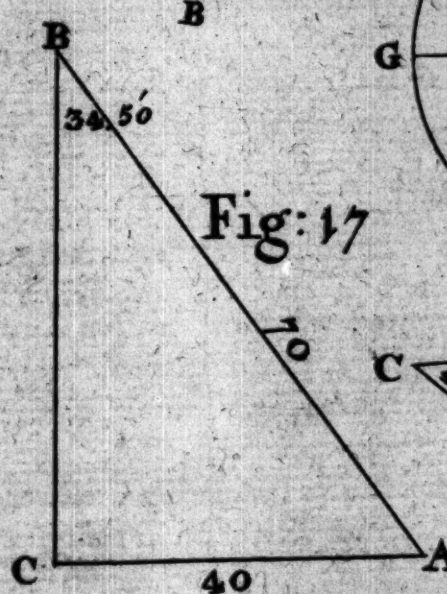


Fig: 17

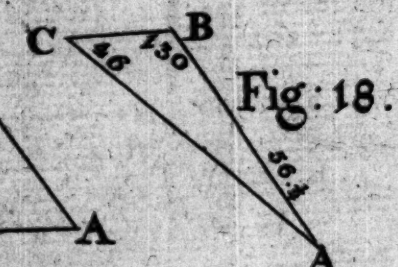


Fig: 18.

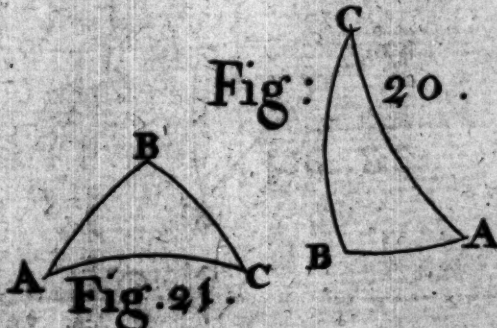


Fig: 20.

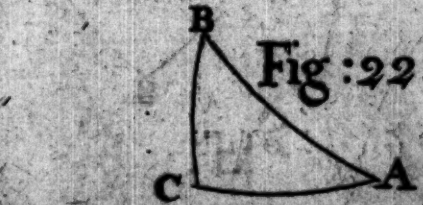


Fig: 22.

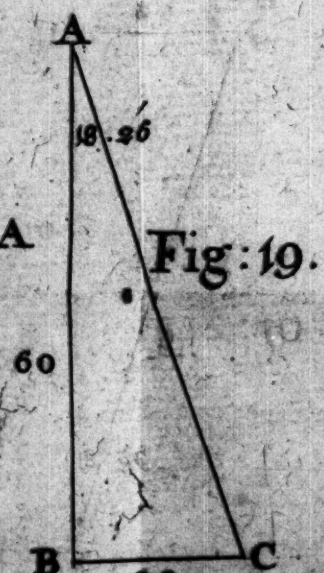


Fig: 19.

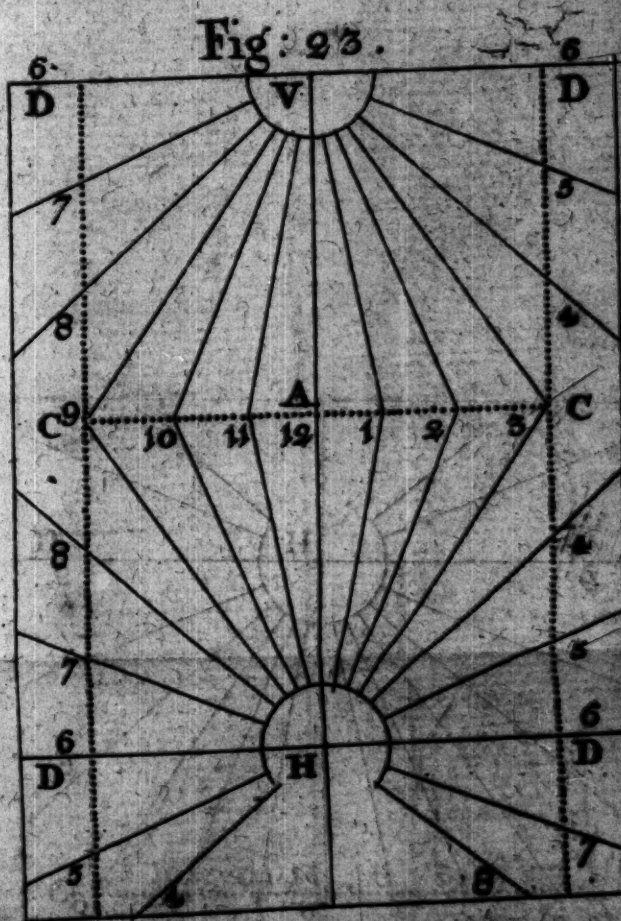


Fig: 23.

Note, This cannot be done unless by several Tryals of opening the Sector, and moving the Points of the Compasses.

Example. Let A and B, be the two given Lines. Lay B from the Center A of the Sector, which suppose to reach to B and B; then take A in your Compasses, and open the Sector so that DD being equal to A, AD may be equal to BB. And then AD or BB will be a Mean Proportional between the Lines A and B; for as AB is to BB, so is AD, equal to BB, to DD. Fig. 13.

P R O B. VIII.

To divide a Line in the same Manner as another Line is already divided.

First, Take the given Line already divided, and laying it on both Sides of the Sector from the Center, note how far it extendeth. Then take out the second Line, which is to be divided, and to it open the Sector in the Terms of the first Line. This done, take out the Parts of the first Line, and place them also on the same Side of the Sector from the Center. And the Parallels taken in the Terms of these Parts, will be the correspondent Parts in the Line to be divided.

Example. Let AB be a Line divided in D and E, and BC the Line to be divided in such manner as AB is. Fig. 14.

First, Take out the Line AB, and place it on the Lines of Lines in AB, AC, both from
E
the

the Center A; then take out the second Line BC, and to it open the Sector in B and C, the Terms of the first Line. The Sector remaining thus opened, take out AD, and AE, the Parts of the first Line AB, and place them also on both sides of the Sector in AD, AE; and the Parallel DD, gives BF, and the Parallel EE, BG. Now the Line BC, is divided in F and G, as the Line AB, is in D, E; which is the thing requir'd.

P R O B. IX.

To open the Sector so, that the two Lines of equal Parts may make a right Angle.

Chuse three Numbers that may express the Sides of a right angled Triangle; as are, for example, the Numbers 3, 4, 5, or their Multiples; but because it is better to take them bigger, chuse 60, 80, and 100. Now take in your Compasses the Distance from the Center of the Sector on either of the Lines of Lines to 10, which let represent 100; then open the Sector so, that the Distance between 6 on one Line of Lines, and 8 on the other, (the one representing 60, and the other 80) be equal to that Distance: And then the Sector will be opened so that the two Lines of Lines make a right Angle.

PROB.

P R O B. X.

To find a right Line equal to the Circumference of a given Circle.

The Diameter of a Circle to its Circumference, is as about 3.18 to 10; whence take the Diameter of the proposed Circle between your Compasses, and set it over as a Parallel on 3.18 and 3.18 of the Lines of Lines. The Sector remaining thus opened, take the parallel Extent of 10 and 10, and that will give you near the Length of the Circumference sought.

P. R O B. XI.

To inscribe a regular Polygon in a given Circle.

Take the Radius of the given Circle between your Compasses, and set it over as a Parallel on 6 and 6 of the Lines of Polygons; then the parallel Distance of the Numbers representing the Number of Sides a Polygon is to have, will be the Side of the requir'd Polygon.

Example. Suppose AGH be a Circle, in which it is required to *Fig. 15.* inscribe a Pentagon. Take the Radius AB in your Compasses, and set it over as a Parallel on 5 and 6 of the Lines of Polygons; then the parallel Distance of 5 and 5, on the said Lines of Polygons, will give AC, the Side of a Pentagon to be inscribed in the Circle. Whence it will be easie to make the Polygon in carrying the Distance AC round the Circle, and drawing 5 right Lines,

P R O B. XII.

Upon a right Line given to make a regular Polygon.

Take the given right Line between your Compasses, and to it open the Sector in the Points of the Number of Sides the Polygon is to have; then the parallel Distance between 6 and 6, will give the Length of the Radius of a Circle, whose Center is the Vertex of an Isocles Triangle made with the aforesaid Radius's upon the given Line. Now I say, if about the said Center a Circle be described with the aforesaid Radius, and the aforesaid given Line be carried round its Circumference, and right Lines drawn, that the proposed Polygon will be made.

Example. To make a Heptagon *Fig. 16.* upon the given Line AB. Take the said Line between your Compasses, and set it over as a Parallel on the Points of 7 and 7 of the Lines of Polygons; then the parallel Distance between 6 and 6 will give the Radius of a Circle. Now with this Radius setting one Foot in the Points A, B, the Extremes of the given Line, describe two Arc's cutting each other in C, about which describe the Circle ABD; then carry AB round the Circumference, and draw 6 right Lines, and the Heptagon will be made.

C H A P. V.

Of working Proportions in Lines and Sines, Lines and Tangents, Sines and Tangents, &c. exemplified, in the Resolution of some of the Cases of plain, and spherical Trigonometry.

P R O B. I.

The Hypothenuſe, and Baſe of a right lined right angled Triangle being given, to find the Angle oppoſite to the Baſe.

Suppoſe the Length of the Hypothenuſe AB be 70 Miles, and *Fig. 17.* the Length of the Baſe AC 40 Miles, what is the Quantity of the Angle ABC.

As the Length of the Hypothenuſe AB is to Radius, ſo is the Baſe AC to the Sine of the Angle ABC.

Therefore to work this Proportion on the Sector, take the Extent from the Center on either of the Lines of Lines to the Number 7, which let represent the Length of the Hypothenuſe 70 Miles; then open the Sector, and ſet this Extent over as a Parallel on the Points of 90 and 90 on the Line of Sines. The Sector remaining thus opened, take the Length of the Baſe 40 Miles from the Line of Lines, and applying it parallelwiſe on the Sines, its Extremes will give 34 Degrees 50 Minutes for the Quantity of the Angle ABC ſought.

PROB.

P R O B. II.

The Base AC of the oblique angled Triangle ABC being given, suppose 60 Leagues; the Angle ABC, 150 Degrees, and the Angle ACB, 46 Degrees; to find the Length of the Side AB.

As the Sine of the Angle ABC is to the Length of the Base AC, so is the Sine of the Angle ACB to the Length of the Side AB sought.

To work this by the Sector, take the lateral Extent of 46 Degrees on either of the Lines of Sines; then open the Sector, and apply this Extent parallelwise in the Points of 30 and 30, (the Complement of the Angle ABC) on the Lines of Sines. The Sector remaining thus open'd, take the parallel Extent from 6 to 6, representing 60 and 60, on the Lines of Lines, and measure it laterally on either of them, and you will find it to be about 86, which is the Length in Leagues of the sought Side AB.

P R O B. III.

The Perpendicular and Base of a right angled Triangle ABC being given, the first suppose 20 Yards, and the latter 60; to find the Quantity of the Angle CAB.

As the Base AB is to the Perpendicular BC, so is the Radius to the Tangent of the Angle BAC sought.

To

To do this by the Sector, take 20 from your Line of Lines, represented by 2; then set this over as a Parallel on 6 and 6, representing 60, on the said Lines of Lines. The Sector remaining thus open'd, take the parallel Radius, that is, the Extent from 45 Degrees to 45 Degrees on the Lines of Tangents, and this laterally measur'd on either of the Lines of Tangents, will give about 18 Degrees 26 Minutes, for the Quantity of the sought Angle CAB.

Note, If the parallel Radius be longer than either of the Lines of Tangents, which it will often be, then you must make use of the Lines of lesser Tangents. But these lesser Tangents must make the same Angle with each other as the Lines of Lines do, as I mentioned before in the Description of the Sector; for otherwise the Proportion cannot be worked, because similar Triangles are requir'd.

P R O B. IV.

The Hypothenuse AC, and the Angle ACB, of the right angled spherical Triangle ABC, being given, Fig. 20. the one 50 Degrees, and the other 23 Degrees 30 Minutes; to find the Side AB.

As Radius is to the Sine of the Hypothenuse, so is the Sine of the Angle ACB to the Sine of the sought Side AB.

To do this by the Sector, take the lateral Extent of 50 Degrees on the Sines between your Compasses; then open the Sector, and set over

over this Extent on 90 and 90 of the Lines of Sines. The Sector remaining thus opened, take the parallel Extent of 23 Degrees 30 Minutes, on the Sines, which laterally measur'd on either of them, will give the Sine of about $17\frac{1}{2}$ Degrees, the sought Side AB.

P R O B. V.

The Angles BCA, BAC, and the Side BC of an oblique spherical Triangle, being given; the first 20 Degrees, the second 30 Degrees, and the last 25 Degrees; to find the Side AB.

As the Sine of the Angle BAC is to the Sine of the Side BC, so is the Sine of the Angle BCA to the Sine of the Side AB sought.

To do this by the Sector; open it, and set the Sine of 25 Degrees over as a Parallel on the Sines of 30 and 30; then the Sector remaining thus opened, take the parallel Sine of 20 Degrees, and measure it laterally on the Sines, and you will find it almost 17 Degrees 50 Minutes, which will be the Quantity of the sought Side AB.

P R O B. VI.

The Side AC, and the Angle BAC of the right angled spherical Triangle ABC being given; the one 27 Degrees 54 Minutes, and the other 23 Degrees 30 Minutes; to find the Side BC.

As Radius is to the Sine of the given Side AC,

AC; so is the Tangent of the given Angle BAC, to the Tangent of the required Side BC.

To do this by the Sector; open it, and set over the Sine of 27 Degrees 54 Minutes, as a Parallel on 90 and 90 of the Lines of Sines. The Sector remaining thus opened, take the parallel Tangent of 23 Degrees 30 Minutes, and measuring it laterally on either of the Lines of Tangents, you will find it about 11 Degrees 30 Minutes, which is the Quantity of the sought Side BC.

These few Examples being well understood; any one may of himself find out the manner of working any other of the Cases of plain or spherical Trigonometry, by the Sector. Therefore I now proceed to shew the excellent Use of this Instrument, in drawing the Hour Lines upon an Horizontal, and erect South Dial.

P R O B. VIII.

To draw the Hour Lines upon an Horizontal and Vertical Plan.

First, draw a right Line AC for *Fig. 23.* the Horizon and Equator, and cross it at the Point A, about the middle of the Line, with AB another right Line, which may serve for the Meridian and Hour Line of 12; then take out 15 Degrees from your Tangents, and lay them off in the Equator on both sides from 12; one of which Points will serve for the Hour of 11, and the other for the Hour of 1. Again take out the

F

Tangent

Tangent of 30 Degrees, and prick it down in the Equator on both sides from 12; one of which Points will serve for the Hour of 10, and the other for the Hour of 2. In like manner prick down the Tangent of 45 Degrees for the Hours of 9 and 3, and the Tangent of 60 Degrees for the Hours of 8 and 4, and the Tangent of 75 Degrees for the Hours of 7 and 5.

In like manner may the Parts of an Hour be pricked down, in allowing 7 Degrees 30 Minutes for every half Hour, and 3 Degrees 45 Minutes for every Quarter. This done, you are to consider the Latitude of the Place, and the Quality of the Plan; for the Secant of the Latitude will be the Semidiameter in a vertical Plan, and the Secant of the Complement of the Latitude in an horizontal Plan.

For Example; About *London* the Latitude is 51 Degrees 32 Minutes, and let the Plan be vertical. If you take AV, the Secant of 51 Degrees 32 Minutes from the Sector, having first open'd the Sector so, that the Radius of the Lines of Secants be equal to the Radius of the Tangents pricked off, and prick it down in the meridian Line from A to V; the Point V will be the Center of the Dial: And if you draw right Lines from V to 11, and 10, and the rest of the Hour Points, these will be the Hour Lines required.

But if the Plan be an Horizontal one, you must take out AH, the Secant of 38 Degrees 28 Minutes, and prick it down in the Meridian
Line

Line from A to H; so right Lines drawn from the Center H to the Hour Points, will be the Hour Lines required. Only the Hour of 6 is wanting, which must always be drawn parallel to the Equator, thro' the Center V, in a vertical Plan; and thro' the Center H, in an horizontal Plan.

But if it happen that some of the Hour Points fall out of your Plan, you may remedy your self, both in the vertical and horizontal Plans.

For if at the Hour Points of 3 and 9 you draw occult Lines parallel to the Meridian, the Distances DC, between the Hour Line of 6, and the Hour Points of 3 and 9, will be equal to the Semidiameter AV, in a vertical, and AH, in an horizontal Plan; and if they be divided in the same manner that the Line AC is divided, you will have the Points of 4, 5, 7, and 8, with their Halves and Quarters.

As in the horizontal Plan, take out the Semidiameter AH, and make it a parallel Radius, by setting it over in the Sines of 90 and 90: Then take the parallel Tangent of 15 Degrees, and it will give you the Distance from 6 to 5, and from 6 to 7, in your horizontal Plan. The Sector remaining thus opened, take out the parallel Tangent of 30 Degrees, and it will give you the Distance from 6 to 4, and from 6 to 8, in your horizontal Plan. The like may be done for half and quarters of Hours.

C H A P. VI.

Of the Use of the Lines of Numbers, artificial Sines, and Tangents.

S E C T I O N I.

Of the Use of the Line of Numbers.

THE Order of the Divisions on the Line of Numbers is thus; it begins with 1, and so proceeds with 2, 3, 4, 5, 6, 7, 8, 9; and then 1, 2, 3, &c. to 10 at the end. Each of these Divisions are subdivided into 10 Parts, and every of these 10th Parts from 1 in the middle to 2, are again subdivided into 5 Parts. But the 10th's from the second 2 to 5, are only subdivided into two Parts.

Note, Each 10th should have been subdivided into 10 other Parts, called Centesmes, but that the Length of the Line will not admit of it.

Numeration on the Line of Numbers.

P R O B. I.

Any whole Number under four Figures being given; to find the Point on the Line of Numbers representing the same.

First seek for the first Figure of your Number amongst the long Divisions that are figured, and that leads you to the first Figure of the Number. Then for the second Figure, count so many Tenth's from the aforesaid Division

vision forwards, as the second Figures amount to ; then for the third Figure, count from the last Tenth, as many Centesmes as the third Figure contains ; and for the fourth Figure, count from the last Centesme, so many Millions, as the fourth Figure hath Unites ; and that will be the Point where the Number is on the Line of Numbers.

Example ; To find the Point on the Line of Numbers representing the Number 12. Now because the first Figure of this Number is 1, you must take the 1 in the middle for the first Figure ; then the next Figure being 2, count the Tenths from that 1, and there will be the Point representing 12.

Again ; To find the Point representing 144. First, as before, take for 1, the first Figure of the Number 144, the middle Figure 1 ; then for the second, (*viz.* 4.) count 4 Tenths forwards ; lastly for the other 4, count 4 Centesmes further, and that is the Point for 144.

Lastly ; To find the Point representing 1728. First, as before, for 1000 take the middle 1 ; secondly for 700 reckon 7 Tenths forwards ; thirdly for 20 reckon 2 Centesmes from that seventh Tenth ; and lastly, for the 8 you must reasonably estimate the following Centesme to be subdivided into 10 Parts, and 8 of them will be the Point on the Line of Numbers representing 1728. Understand the same for any other Number. The aforesaid Point may also represent .1728, or 1.728, or 17.28, or lastly 172.8.

PROB.

P R O B. II.

One Number being given to be multiply'd by another; to find the Product.

Extend your Compasses from Unity to the Multiplier; then the same Extent apply'd the same way, from the Multiplicand, will cause the movable Point to fall upon the Product.

Example; Let 6 be given to be multiply'd by 5: Extend your Compasses from 1 to 5, and the same Extent will reach from 6 to 30, the Product sought. Again, suppose 125 is to be multiply'd by 144: Extend your Compasses from 1 to 125, and the movable Point will fall from 144 upon the Product 18000.

P R O B. III.

One Number being given to be divided by another; to find the Quotient.

Extend your Compasses from the Divisor to 1; and the same Extent will reach from the Dividend to the Quotient. Or, extend the Compasses from the Divisor to the Dividend, the same Extent will reach the same way from 1 to the Quotient.

Example; Let 750 be a Number given to be divided by 25. Extend your Compasses downwards from 25 to 1; then applying that Extent the same way from 750, and the other Point will fall upon 30, the Quotient sought.

P R O B. IV.

Three Numbers being given; to find a fourth Number proportional to them.

Extend your Compasses from the first of the given Numbers to the second of the same Denomination; and this Extent apply'd the same way, will reach from the second to the fourth Proportional sought.

Example; If 8 give 16, what will 12 give? Extend your Compasses from 8 to 16; and this Extent apply'd the same way will reach from 12 to 24, the fourth Proportional sought.

Again, If 38 gives 76, what will 96 give? Extend your Compasses from 38 to 76, and the same Extent will reach from 76 to 192, the fourth Proportional sought.

P R O B. V.

To find a mean Proportional between two given Numbers.

Bisect the Distance between the given Numbers, and that Point of Bisection will fall on the mean Proportional sought.

Example; The Extremes being 8 and 32, the middle Point between them will be 16, the mean Proportional sought. For as 8 is to 16, so is 16 to 32.

P R O B. VI.

To find the Square Root of a given Number.

The square Root of any Number is always a mean Proportional between 1 and the said Number;

Number; so that to find the square Root of a Number is only to take half the Distance between 1 and the Number given, and that Point of Bisection will be the square Root of the Number given. But if the Figures of the Number be even, you must look for the Unite at the beginning of the Line, and the Number in the second Part, and the Root in the first Part. Or rather reckon 10 at the end to be Unity, and then both Root and Square will fall backwards towards the middle, in the second Part of the Line. But if the Number be odd, then the middle Unite will be most convenient, and both Root and Square will be found from thence forwards towards 10.

SECTION II.

Of the conjoint Use of the Lines of Numbers, artificial Sines, and Tangents.

These Lines of artificial Numbers, Sines, and Tangents, are so contrived, and set to each other, that by means of a Pair of Compasses, any Problem, whether in right lined or spherical Trigonometry, may be very expeditiously solved by them with tolerable Exactness.

The Sines are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, signifying Degrees. Every one of the Degrees, as far as 6, are subdivided into 10 Parts, signifying every 6 Minutes; then the Degrees from 6 to 10, are only subdivided into 6 Parts for every 10 Minutes. Again, each of the 10 Degrees, which

which have not Numbers set to them, for avoiding Confusion, between 10 and 20, are only divided into 4 Parts, for every 15 Minutes; the Degrees between 20 and 30, are divided into 3 Parts, for every 20 Degrees; the Degrees from 30 to 50, are halved, for every 30 Minutes; after which to 70 there are only Degrees, and from 70 to 80 there are only every two Degrees represented; and from 80 to 90 none.

The Line of artificial Tangents, is number'd 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 45. To the Tangents 10, 20, 30, 40, are set their Complements 80, 70, 60, 50. Also these signify Degrees, like as the Sines.

These Degrees are each subdivided, to the Tangent of 30 Degrees, into the same Number of Parts, as those on the artificial Sines. Each of which represent the same Numbers of Minutes as those on the Sines. But from 30 Degrees to 45, each Degree is subdivided into 3 Parts, for every 20 Minutes.

P R O B. I.

To solve the Case and Example of Prob. I. of the last Chapter.

Set one Foot of your Compasses upon 7 of the Line of Numbers, representing 70 Miles, and extend the other to 40 Miles, represented by 4. The Compasses remaining thus opened, set one Foot upon 90 of the Line of Sines, and the other Foot will fall downwards upon the Line of Sines, on the Sine of 34 Degrees

grees 50 Minutes, the Quantity of the Angle sought.

P R O B. II.

To solve the Case and Example of Prob. II. of the last Chapter.

Set one Foot of your Compasses upon the Sine of 30 Degrees, and extend the other to 60 Leagues on the Line of Numbers: The Compasses remaining thus opened, set one Foot upon the Sine of 46 Degrees, and the other will fall upwards on the Line of Numbers upon 86½ Leagues, the Length of the sought Side.

P R O B. III.

To solve the Case and Example of Prob. III. of the last Chapter.

Set one Foot of your Compasses on 60 of the Line of Numbers, and extend the other to 45 Degrees of the Line of Tangents: The Compasses remaining thus opened, set one Foot upon the Number 20 on the Line of Numbers, and the other will fall on the Line of Tangents upon the Tangent of about 18 Degrees 26 Minutes, the Quantity of the Angle sought.

P R O B. IV.

To solve the Case and Example of Prob. IV. of the last Chapter.

Set one Foot of your Compasses upon the Sine of 50 Degrees, and extend the other to the Radius or Sine of 90 Degrees; the Compasses

passes remaining thus opened, set one Foot upon the Sine of 23 Degrees 30 Minutes, and the other will fall downward upon the Sine of 17 Degrees, and about 3 Fourths, for the Length of the sought Side.

P R O B. V.

To solve the Case and Example of Prob. V. of the last Chapter.

Set one Foot of your Compasses upon the Sine of 30 Degrees, and extend the other downwards to the Sine of 25 Degrees: The Compasses remaining thus opened, set one Foot upon the Sine of 20 Degrees, and the other will fall upon the Sine of about 16 Degrees, 40 Minutes, the Length of the sought Side.

P R O B. VI.

To solve the Case and Example of Prob. VI. of the last Chapter.

Set one Foot of your Compasses upon the Sine of 90 Degrees, and extend the other downwards to the Sine of 27 Degrees, 54 Minutes: Your Compasses remaining thus opened, set one Foot upon the Tangent of 23 Degrees, 30 Minutes, and the other Foot will fall downwards upon the Tangent of about 11½ Degrees, the Quantity of the sought Side.

Note, If in working Proportions in Sines and Tangents, the Foot of your Compasses should fall beyond the Tangent of 45 Degrees,

grees; as it will often happen, then you must take a Point as far on this Side of 45 Degree, as the said Point of your Compasses falls beyond 45 Degrees, and that Point will give you the Tangent of the Complement to 90 Degrees, of the Side or Angle sought.

Example: Suppose this Proportion was to be worked as the Sine of 20 Degrees is to Radius; so is the Tangent of 30 Degrees to another Tangent; set one Foot of your Compasses on the Sine of 20 Degrees, and extend the other to 90 Degrees: Your Compasses remaining thus opened, set one Foot upon the Tangent of 30 Degrees, and the other will fall upwards beyond the Tangent of 45 Degrees; then finding how far it falls beyond, with one Foot of your Compasses on 45 Degrees, set off that Extent on this side 45 Degrees, and you will find it to fall upon the Tangent of the Complement of the sought Tangent, *viz.* on 30 Degrees, 40 Minutes; whence the Tangent sought is 59 Degrees, 20 Minutes.

Hence you see, that in working Proportions in Numbers and Sines, or Sines and Tangents, it is but setting one Foot of your Compasses upon the Number, Sine, or Tangent, in the first Term, and extending the other to the second Term of a contrary Denomination; and then that Extent will reach from the third Term to a fourth sought.